PROBLEMS
Sections 16.1-16.4

16.1. (I) (a) Calculate the frequency range of visible light, given its wavelength range to be 380-770 nm. (b) Calculate the ratio of the highest to lowest frequency the eye can see. (c) Calculate the ratio of the highest to lowest frequency sounds that the human ear can hear. (d) Compare the two ratios. Which of the ratios is larger?

16.1a) Calculate the frequency: The speed of light c equals the frequency times the wavelength. The wavelength of light is often measured in units called nanometers. A nanometer is 10^{-9} meters.

Baby step 1: Convert 380 nm to meters.

$$380nm\frac{1x10^{-9}\,meter}{1nm} = 380x10^{-9}\,meters$$

Baby step 2: Solve $c = f\lambda$ for frequency.

$$c = f\lambda;\ \text{Therefore}\ f = \frac{c}{\lambda}$$

Baby step 3: Substitute values and determine the frequency.

$$f = \frac{c}{\lambda} = \frac{3x10^{8}\,\frac{m}{s}}{380x10^{-9}\,m} = \frac{7.89x10^{14}}{\text{sec}}$$

The shortest wavelength of visible light is 380 nm. Its frequency is $7.89x10^{14}\,Hz$. That will be visible light's highest frequency.

The longest wavelength of visible light is 770 nm. Converting 770 nm to meters and solving for frequency, we get:

$$f = \frac{c}{\lambda} = \frac{3x10^{8}\,\frac{m}{s}}{770x10^{-9}\,m} = \frac{3.90x10^{14}}{\text{sec}}$$

Therefore, the lowest frequency visible light will be $3.90x10^{14}\,Hz$

Ans.

Visible light frequencies range from 3.90 x10^{14} Hz to 7.89 x 10^{14} Hz.

16.1b) Calculate the ratio of the highest to lowest frequency the eye can see.

Ans. Light $\frac{\textit{highest} \text{ freqency}}{\textit{lowest} \text{ freqency}} = \frac{7.89x10^{14}}{3.90x10^{14}} = 2.02$

16.1c) Calculate the ratio of the highest to lowest frequency sounds that the human ear can hear.

Ans. Sound $\frac{\textit{highest} \text{ freqency}}{\textit{lowest} \text{ freqency}} = \frac{20{,}000}{20} = 1{,}000$

16.1d) Compare the two ratios. Which of the ratios is larger?

$$\frac{\text{Range of sound frequencies}}{\text{Range of visible light frequencies}} = \frac{1000}{2.02} \simeq 500$$

Ans. The range of audible sound frequencies is 500 times that of the range of visible light frequencies.

16.2 (I) What is the wavelength of FM radio transmission waves having a frequency of 102 MHz?

$c = f\lambda$; solving for wavelength yields: $\lambda = \frac{c}{f} = \frac{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}}{\frac{102 \text{ x } 10^6}{\text{sec}}} = 2.94 \text{ m}$

16.3 (I) Two microwave frequencies are authorized for use in microwave ovens: a) 900 and b) 2560 MHz. Calculate the wavelength of each.

a) For a frequency of 900 Hz, the wavelength is:

$$c = f\lambda; \text{ therefore } \lambda = \frac{c}{f} = \frac{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}}{900 \text{ x } 10^6/\text{sec}} = 0.33 \text{ meters}$$

b) For a frequency of 2560 MHz, the wavelength is:

$$c = f\lambda; \text{ therefore } \lambda = \frac{c}{f} = \frac{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}}{2560 \text{ x } 10^6/\text{sec}} = 0.117 \text{ meters}$$

16.4 (I) EM radiation having a wavelength of 10 μm (10,000 nm) is classified as infrared radiation. What is its frequency?

$$\lambda = 10\mu m = 10 \times 10^{-6} m$$

$$c = f\lambda;\ f = \frac{c}{\lambda} = \frac{3 \times 10^{8} \frac{m}{sec}}{10 \times 10^{-6} m} = \frac{3 \times 10^{13}}{sec}$$

16.5 (I) What is the smallest possible detail observable (in theory) with a microscope that uses ultraviolet light having a frequency of 1.5 x 10^{15} Hz?

The amount of detail that can be seen is determined by the wavelength of the signal. We know that the speed of light c equals 3 x 10^{8} and we are given frequency is 1.5 x 10^{15}/sec.

$$c = f\lambda;\ \lambda = \frac{c}{f} = \frac{3 \times 10^{8} \frac{m}{sec}}{\frac{1.5 \times 10^{15}}{sec}} = 2 \times 10^{-7} m = 200 \text{ nm}$$

(Note: With modern imaging techniques it is now possible to reduce the limit of resolution down to 100 nm or less).

16.6. (I) Radar systems can be used to detect the shape and size of objects, such as aircraft or geological terrain. If a radar system uses microwaves of frequency 500 MHz, what is the smallest detail that it can detect?

I*n other words, determine the wavelength of light that has a frequency f = 500 x 10^{6}/sec.*

$$c = f\lambda;\ \lambda = \frac{c}{f} = \frac{3 \times 10^{8} \frac{m}{sec}}{\frac{500 \times 10^{6}}{sec}} = 0.6 m$$

16.7 (I) The ideal size of a broadcast antenna with one end on the ground is one-fourth the wavelength of the EM radiation being broadcast. Other sizes will work, but an antenna of size about 1/4 the wavelength is most efficient. Suppose you read that a new radio station has an antenna 50 m high. a) What wavelength light does the tower transmit? b) What frequency does this new station broadcast, assuming that it is the ideal height? c) Is the station AM or FM?

16.7a. *What wavelength light does the tower transmit? We are told that the length of the of an ideal antenna is L = 1/4 the wavelength of the photon and that the antenna is 50 meters high.*

The length L of the tower 1/4 wavelength.

$L = \frac{1}{4}\lambda$; Therefore $\lambda = 4 \bullet L = 4 \bullet 50m = 200m$

Ans.a) The wavelength of the light is 200 meters.

16.7b. *What frequency does this new station broadcast, assuming that it is the ideal height?*

Solve $c = f\lambda$ for frequency and substitute the known and given values.

$$c = f\lambda;\ \ f = \frac{c}{\lambda} = \frac{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}}{200 \text{ m}} = \frac{1.5 \text{ x } 10^6}{\text{sec}} = 1.5x10^6 \text{Hz}$$

16.8 (I) Radar is used to determine distances to various objects by measuring the round-trip time of an echo from the object. Calculate the echo times for the following: (a) a car 100 m from the transmitter; (b) an airplane 10 km from the transmitter; and (c) the planet Venus when it is 1.50 x 10^8 km from earth.

An echo is produced when sound goes out to an object like a wall or the side of a mountain and reflects back to the source. Therefore the object distance is only 1/2 the distance traveled by the echo.

$$\text{Distance} = \overline{\text{V}}\text{t};\ \text{t} = \frac{\text{Distance}}{\overline{\text{V}}}$$

$$\text{a. t} = \frac{200 \text{ m}}{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}} = 6.67\text{x } 10^{-7} \text{sec}$$

$$\text{b. t} = \frac{20 \text{ x } 10^3 \text{ m}}{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}} = 6.67\text{x } 10^{-5} \text{ sec}$$

$$\text{c. } \frac{3 \text{ x } 10^{11} \text{ m}}{3 \text{ x } 10^8 \frac{\text{m}}{\text{sec}}} = 1 \text{ x } 10^3 \text{ sec} = 1000 \text{ sec}$$

16.9 (I) (a) Calculate the energy in electron volts of an infrared photon of frequency 1.0 x 10^{13} Hz. (b) Calculate the energy in electron volts of an ultraviolet photon of frequency 2.0 x 10^{16} Hz. (c) Compare both of these energies with the 5.0 eV needed to disrupt a certain compound and comment on their likely effects.

The energy of the photon is found by: E = hf. Also note that the electron-volt is a unit of energy. It is used when very small amounts of energy are involved. 1 eV = 1.6 x 10^{-19} joules.

16.9.a) Calculate the energy in electron volts of an infrared photon of frequency 1.0 x 10^{13} Hz.

Baby step 1. Calculate the energy in joules.

$$E = hf = 6.626x10^{-34}\,J \cdot s \bullet \frac{1x10^{13}}{s} = 6.626x10^{-21}\,joules$$

Baby step 2. Convert the energy units from joules to eV.

$$6.626x10^{-21}\,joules \bullet \frac{1eV}{1.6x10^{-19}\,joules} = 0.0414eV$$

16.9.b) Calculate the energy in electron volts of an ultraviolet photon of frequency 2.0 x 10^{16} Hz.

$$\text{E} = \text{hf} = 6.626 \text{ x } 10^{-34}\,\text{J} \cdot \text{s} \left(\frac{2 \text{ x } 10^{16}}{\text{sec}} \right) = 1.33 \text{ x } 10^{-17}\,\text{joules}$$

$$1.33 \text{ x } 10^{-17}\,\text{joules}\left(\frac{1 \text{ eV}}{1.6 \text{ x } 10^{-19}\,\text{joules}} \right) = 82.8 \text{ eV}$$

16.9.c) Compare both of these energies with the 5.0 eV needed to disrupt a certain compound and comment on their likely effects.

Ans. The infrared photon cannot disrupt the compound. It only has 0.0414 eV of energy. The ultraviolet photon has much more energy (82.8 eV) than what is needed to disrupt the compound (5eV).

16.10. (I) Calculate the energy in electron volts of a a) a radio photon of frequency 8.0 x 10^5 Hz and (b) a gamma ray of frequency 2.5 x 10^{20} Hz. Compare both of these energies with the 5.0 eV needed to disrupt a certain compound and comment on their likely effects.

The energy of the photon is found by: E = hf. Also note that the electron-volt is a unit of energy. It is used when very small amounts of energy are involved. 1 eV = 1.6 x 10^{-19} joules.

a. The energy of a radio wave with frequency of 8 x 10^5 Hz:

$$E_{photon} = hf = 6.626x10^{-34}\ j \cdot s \bullet \frac{8.0x10^5}{s} = 5.30x10^{-28}\ joules$$

Converting from joules to electron-volts we get:

$$5.30x10^{-28}\ j \bullet \frac{1eV}{1.6x10^{-19}\ j} = 3.31x10^{-9}\ eV$$

This is tiny amount of energy. It is much less than the energy required to disrupt cell function.

b. The energy of a gamma ray with frequency of 2.5 x 10^{20} Hz:

$$E_{photon} = hf = 6.626x10^{-34}\ j \cdot s \bullet \frac{2.5x10^{20}}{s} = 1.66x10^{-13}\ joules$$

Converting from joules to electron-volts we get:

$$1.66x10^{-13}\ j \bullet \frac{1eV}{1.6x10^{-19}\ j} = 1.04x10^{6}\ eV$$

If you are a cell, this is a very large amount of energy. Watch out for those gamma rays. Arm the photon torpedos.

16.13. (II) Suppose you want to see submicroscopic details the size of atoms, about $1x10^{-10}$ m. This distance is the definition of an angstrom. You need to use EM radiation of wavelength $1x10^{-10}$ m or less. (a) Calculate the frequency of EM radiation of wavelength $1x10^{-10}$ m. (b) What is the energy in electron volts of a photon of this radiation? Note that since this energy is far more than is needed to disrupt the atom you observe, the measurement disrupts the object being observed.

a) Calculate the frequency of EM radiation that you will need.

$c = f\lambda$; Solving for frequency and substituting values:

Ans. $c = f\lambda$; therefore $f = \frac{c}{\lambda} = \frac{3x10^{8}\frac{m}{s}}{1x10^{-10}m} = 3x10^{18}\,Hz$

b) What is the energy in electron volts of a photon of this radiation?

The energy of a photon can be calculated using: $E = hf$

$$E = hf = 6.626x10^{-34}\,j \cdot s \bullet \frac{3x10^{18}}{s} = 1.99x10^{-15}\,joules$$

Converting the energy to electron volts:

Ans. $1.99x10^{-15}\,joules \bullet \frac{1ev}{1.6x10^{-19}\,joules} = 1.24x10^{4}\,eV$

16.14. (II) The smallest details observable using EM radiation as a probe have a size of about one wavelength. (a) What is the smallest detail observable with ultraviolet photons of energy 10 e V? (b) What is the smallest detail observable with x-ray photons of energy 100 keV? Convert your answer to nanometers.

a) ***Baby step 1***: *We are given the energy of a photon, in kiloelectron-volts and asked to determine the wavelength. The first step will be to convert the energy to joules.*

$$10eV\frac{1.6x10^{-19}\,joules}{1eV} = 1.6x10^{-18}\,joules$$

Baby step 2: We need an energy equation that has wavelength in it.

$E_{photon} = hf$. Remember that the frequency of the photon is $f = \frac{c}{\lambda}$

Substituting for f, $E_{photon} = hf = \frac{hc}{\lambda}$

Baby step 3: Solve $E_{photon} = \frac{hc}{\lambda}$ for wavelength and substitute values.

$$E_{photon} = \frac{hc}{\lambda};\ \text{Threfore}\ \lambda = \frac{hc}{E} = \frac{6.626x10^{-34}\,j \cdot s \bullet 3x10^{8}\,\frac{m}{s}}{1.6x10^{-18}\,j} = 1.24x10^{-7}\,meters$$

Ans. The smallest detail that can be observed using 10eV EM is 124 nanometers.

14. b) What is the smallest detail observable with x-ray photons of energy 100 keV?

Baby step 1: *We are given the energy of a photon, in kiloelectron-volts and asked to determine the wavelength. The first step will be to convert the energy to joules.*

$$100{,}000eV \bullet \frac{1.6x10^{-19}\,joules}{1eV} = 1.6x10^{-14}\,joules$$

Baby step 2: Substitute values into the previously derived equation for Energy.

$$E_{photon} = \frac{hc}{\lambda};\ \text{Threfore}\ \lambda = \frac{hc}{E} = \frac{6.626x10^{-34}\,j \cdot s \bullet 3x10^{8}\,\frac{m}{s}}{1.6x10^{-14}\,j} = 1.24x10^{-11}\,meters$$

Ans. The smallest detail that can be observed using 10eV EM is $1.24x10^{-11}\,meters$ which is equal to 0.0124 nanometers.

16.16. (III) Calculate the kinetic energies in electron volts of an electron ejected from a material with a work function ϕ = 1.80 eV by a photon of wavelength 400 nm. (b) What is the speed of the ejected electron in meters per second given its mass to be $9.11x10^{-31}kg$.

16.16a Background: *Conservation of energy tells us that the kinetic energy of the ejected electron is equal to the energy of the incoming photon minus the energy required to release the electron from the material. The energy required to release the electron from the material is names the work function and is represented by the greek letter* ϕ. $KE_{electron} = E_{photon} - \phi$

16.16a Baby step 1: *Calculate the energy of the photon in joules.*

The relationship between the energy of the photon and the photon's wavelength is,

$$E_{photon} = \frac{hc}{\lambda}.$$

Substituting values,

$$E_{photon} = \frac{hc}{\lambda} = \frac{6.626x10^{-34}\ j \cdot s \bullet 3x10^{8}\ \frac{m}{s}}{400x10^{-9}\ m} = 4.97x10^{-19}\ joules$$

Baby step 2: *Convert the photons energy from joules to eV.*

$$4.97x10^{-19}\ joules \bullet \frac{1eV}{1.6x10^{-19}\ joules} = 3.11eV$$

Baby step 3: *Determine the kinetic energy of the electron in electron volts.*

$$KE_{electron} = E_{photon} - \phi$$

Ans. $KE_{electron} = 3.11eV - 1.8ev = 1.31eV$

16.16b Background: *In order to determine the speed of the ejected electron, we must convert its energy units from eV to joules. Next we will set the energy equal to the electron's kinetic energy. The mass of an electron is* $9.11x10^{-31} kg$.

Baby step 1: $1.31eV \bullet \frac{1.6x10^{-19}\ joules}{1eV} = 2.10x10^{-19}\ joules$

Baby step 2: $K.E. \equiv \frac{1}{2}mv^2 = 2.10x10^{-19}\ joules$

Ans. $v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \bullet 2.10x10^{-19}\ j}{9.11x10^{-31}\ kg}} = 6.79x10^{5}\ \frac{m}{s}$

16.20. (III) Some older police radar units determine the speed of motor vehicles using a method analogous to the ultrasound Doppler shift technique used in medical diagnostics. Radar is bounced from a moving vehicle, and the returning wave is Doppler shifted. The echo is mixed with the original frequency, producing beats. If the police radar uses microwave frequency of 1.5 x 10^9 Hz and a beat frequency of 150 Hz is obtained, what is the speed of the vehicle?

16.20. Background:

Because an echo is involved, the formula must be modified slightly from the one given in Ch.8. Because of the echo, the Doppler shift is doubled. The equation for the Doppler shift involving echo's is

$$f_D = \frac{2 f_0 V_{scatterer} Cos\theta}{V_{propagaton} - V_{scatterer}}$$

Where;

f_D is the Doppler shift. In this example it is the beat frequency.

θ is the angle of incidence which we will assume to be 0 degrees for this example,

f_0 is the operating frequency of the radar gun.

V_{prop} in this example is the speed of light.

$V_{scatter}$ is the speed of the car.

Baby step 1: Because the speed of the car, $V_{scatter}$ is so slow compared to the speed of light, we do not have to subtract it $V_{propagation}$ in the denominator of our equation. Our equation then becomes,

$$f_D = \frac{2 f_0 V_{scatterer} Cos\theta}{V_{propagaton}}$$

Baby step 2: Solve for $V_{scatterer}$.

$$V_{scatterer} = \frac{f_D \bullet V_{propagation}}{2 \bullet f_0 Cos\theta}$$

Baby step 3: Substitute the values and do the math.

***Ans.** *$$V_{scatterer} = \frac{f_D \bullet V_{propagation}}{2 \bullet f_0 Cos\theta} = \frac{\frac{150}{s} \bullet 3x10^8 \frac{m}{s}}{2 \bullet \frac{1.5x10^9}{s} \bullet Cos(0)} = 15 \frac{m}{s}$$