Section 6.1

6.1. (I) As a woman walks, her entire weight is momentarily supported on the heel of one shoe. Calculate the pressure exerted on the floor by a high-heel shoe worn by a 50-kg woman if the heel has an area of 2.0 cm^2. Express the pressure in a) newtons per square meter and b) atmospheres.

$$\text{Pressure} \equiv \frac{\text{Force}}{Area}$$

a) In this problem, the force is the woman's weight.

Weight = mg = 50 kg(9.8 m/sec^2) = 490 newtons.

Next we have to convert the area of her shoe to square meters.

$$Area = 2cm^2\left(\frac{1m^2}{10{,}000cm^2}\right) = 2x10^{-4}m^2$$

Substituting the values, we get:

Ans.

$$\text{Pressure} \equiv \frac{Force}{Area} = \frac{490N}{2x10^{-4}m^2} = 2.45x10^6\frac{N}{m^2}$$

b) Converting to atmospheres:

$$\text{Ans. } 2.45x10^6\frac{N}{m^2}\cdot\frac{1atm}{1.01x10^5\frac{N}{m^2}} = 24.3 \text{ Atmospheres}$$

6.2. (I) What pressure is exerted by the tip of a nail struck with a force of 20,000 N? Assume the tip is a 1.5-mm-radius circle.

Force = 20,000 newtons.

The area of the tip of the nail is: Area = $\pi r^2 = \pi\,(1.5 \times 10^{-3})^2 = 7.069 \times 10^{-6}$ m^2

$$\textbf{\textit{Ans.}}\ \text{Pressure} \equiv \frac{Force}{Area} = \frac{20{,}000N}{7.06x10^{-6}m^2} = 2.83x10^9\frac{N}{m^2}$$

6.18. A certain ideal hydrometer has uniform density of 0.90 g/cm^3.

a. What percentage of the hydrometer will be submerged if it is places in a fluid of density 0.95 g/cm^3? ***See fig. 6.21.***

$$\%\ submerged = \frac{\rho_{object}}{\rho_{fluid}} \bullet 100$$

$$\%\ submerged = \frac{0.90\frac{g}{cm^3}}{0.95\frac{g}{cm^3}} \bullet 100 = 94.7\%$$

Ans. 94.7% is submerged below the surface of the fluid.

18**b.** What fraction is submerged if it is placed in a fluid of 1.05 g/cm^3 density?

$$\%submerged = \frac{\rho_{object}}{\rho_{fluid}} \cdot 100 = \frac{0.90\frac{g}{cm^3}}{1.05\frac{g}{cm^3}} = 85.7\%$$

Ans. 85.7 % is submerged.

6.19. (I) What is the density of a piece of wood that floats in water with 70% of its volume submerged?

We will use the equation for % submerged. Unless stated otherwise, the density of water is $1.00\frac{g}{cm^3}$. Also note that 70% is $\frac{70}{100} = 0.70$.

$$\%\ submerged = \frac{\rho_{object}}{\rho_{fluid}} \bullet 100$$

Solving for density of object yields,

$$\rho_{object} = \frac{\%\ submerged \bullet \rho_{fluid}}{100} = \frac{70 \bullet 1.00\frac{g}{cm^3}}{100} 0.70\frac{g}{cm^3}$$

Ans. The wood's density is $0.70\frac{g}{cm^3}$

Section 6.5

6.28. (I) Normal blood flow rate in a resting adult is approximately 5.0 liters/min. Convert this to cubic centimeters per second.

By definition, there are 1000 cm^3 in 1.00 liter. We all know that there are 60 seconds in 1 minute. I like to think of unit conversion as multiplying by one. That is, multiplying by a fraction whose numerator and denominator are equal to each other.

$$5\frac{liters}{\text{min}}\bullet\frac{1000cm^3}{1liter}\bullet\frac{1\,\text{min}}{60\,\text{sec}}=83.3\frac{cm^3}{\text{sec}}$$

Ans. Normal blood flow rate in a resting adult is approximately $83.3\frac{cm^3}{\text{sec}}$.

6.29. (I) If the flow rate in an IV setup is 2.0 cm^3/min, how long does it take to empty a 1.0-liter bottle of IV solution?

$$\text{Flow rate}\equiv\frac{Volume}{time}$$

Solving for time yields the following:

$$time=\frac{Volume}{Flow\ \text{rate}}=\frac{1.0liter}{2.0\frac{cm^3}{\text{min}}}$$

notice that we have volume in terms of both liters and cm^3. We need to convert to the same set of volume units.

$$time=\frac{Volume}{Flow\ \text{rate}}=\frac{1.0liter}{2.0\frac{cm^3}{\text{min}}}\bullet\frac{1000cm^3}{1.0liter}=500\ \text{minutes}$$

Ans. It will take 500 minutes to empty the IV setup.

6.30. (I) A glucose solution is being administered intraveneously (in an IV as in Figure 6.9) with a flow rate of 3.0 cm^3/min. If the glucose solution is replaced by blood plasma having a viscosity of 1.5 times that of glucose, what is the new flow rate?

From Poiseuille's law we see that there is an ***inverse*** relationship between the flow rate of a fluid and the fluids viscosity, η.

$$\text{Flow Rate}=\frac{\Delta P\pi r^4}{8\eta L}$$

Therefore, to calculate the new flow rate, we ***divide*** the old flow rate by 1.5.

$$\textit{Ans.}\quad Flow\ \text{rate}'=\frac{\text{Flow rate}}{1.5}=\frac{3\frac{cm^3}{\text{min}}}{1.5}=2\frac{cm^3}{\text{min}}$$

6.31. (II) A fluid is flowing through a tube with a flow rate of 100 cm3/sec. Calculate the new flow rate if each of the following changes are made with all other factors remaining the same as the original conditions: (a) The pressure difference is one-third its original value. (b) The length of the tube is 1.5 times its original length. (c) The radius of the tube is 20% of its original value. (d) A fluid with a viscosity one-half the original is substituted. (e) The radius is 90% of its original value and the pressure difference is 1. 5 times its original value.

$$\text{Flow Rate}=\frac{\Delta P\pi r^4}{8\eta L}$$

a) From Poiseuille's law for Flow Rate, we see that flow rate is directly proportional to Δ P. Because they are ***directly proportional***, we will ***multiply*** the old flow rate by the ratio of the new pressure difference compared to the original pressure difference.

Ans. $\text{Flow rate}' = \frac{1}{3} original \text{ flow rate}=\frac{1}{3}\bullet 100\frac{cm^3}{sec} = 33.3\frac{cm^3}{\text{sec}}$

b) From Poiseuille's law we see that there is an ***inverse*** relationship between the flow rate of a fluid and the length of the tube.

$$\text{Flow Rate}=\frac{\Delta P\pi r^4}{8\eta L}$$

We are told that new tube length is 1.5 times longer than the original length. Because of the ***inverse*** relationship between flow rate and length L, to calculate the new flow rate, we ***divide*** the old flow rate by 1.5.

Ans. $Flow \text{ rate}'= \frac{\text{Flow rate}}{1.5} = \frac{100\frac{cm^3}{s}}{1.5} = 66.7\frac{cm^3}{s}$

c) From Poiseuille's law we see that flow rate is ***directly*** proportional to r^4.

$$\text{Flow Rate}=\frac{\Delta P\pi r^4}{8\eta L}$$

We are told that the tube radius is 20%, 0.20, of its original value. Because they are ***directly proportional***, we will ***multiply*** the old flow rate by the ratio of the new radius to the original radius, raised to the fourth power.

Ans. $Flow \text{ rate}' = \text{original Flow rate}\bullet 0.20^4 = 100\frac{cm^3}{\text{sec}}\bullet 0.20^4 = 0.16\frac{cm^3}{\text{sec}}$

d) We are told that a fluid with a viscosity one-half the original is substituted. From Poiseuille's law we see that there is an ***inverse*** relationship between the flow rate of a fluid and the fluids viscosity, η.

$$\text{Flow Rate} = \frac{\Delta P \pi r^4}{8\eta L}$$

Therefore, to calculate the new flow rate, we ***divide*** the old flow rate by $\frac{1}{2}$.

Ans. $$\text{New Flow rate} = \frac{\text{original flow rate}}{\frac{1}{2}} = \frac{\frac{100cm^3}{\text{sec}}}{\frac{1}{2}} = 200\frac{cm^3}{\text{sec}}$$

e) The new radius is 0.9 of the original radius and the pressure difference is 1.5 times the original value. From Poiseuille's law, we see that the flow rate is ***directly proportional*** to both the radius raised to the fourth power and the pressure difference. Therefore the new flow rate will be the original flow rate multiplied by both 0.90^4 and 1.5.

Ans. $$Flow\ \text{rate}' = 100\frac{cm^3}{\text{sec}} \bullet 0.90^4 \bullet 1.5 = 98\frac{cm^3}{\text{sec}}$$

6.32. (II) A flow rate of 20 cm^3/sec is obtained through a tube of length 100 cm and radius 0.50 cm when there is a pressure difference of 25 cm of water along the tube. Calculate the new flow rate when the following changes are made, assuming all other factors remain the same as the original conditions. (a) A 350-cm long tube replaces the original. (b) The tube radius is decreased to 0.40 cm. (c) The pressure difference is increased to 40 cm of water. (d) The pressure difference is increased to 50 cm of water and the radius is decreased to 0.20 cm.

a) We first must determine the ratio of new length to old.

$$\frac{L'}{L} = \frac{350cm}{100cm} = 3.5$$

From Poiseuille's law we know that flow rate is ***inversely proportional*** to the length of the tube, the new flow rate will equal the original flow rate ***divided*** by 3.5.

Ans. $$Flow\ \text{rate}' = \frac{Flow\ \text{rate}}{3.5} = \frac{20\frac{cm^3}{\text{sec}}}{3.5} = 5.71\frac{cm^3}{\text{sec}}$$

b) First determine the ratio of new radius to old radius.

$$\frac{r'}{r} = \frac{0.40cm}{0.50cm} 0.80$$

Since Flow rate is ***directly proportional*** to r^4, the new flow rate will equal the original flow rate ***multiplied*** by 0.80^4.

$$Flow \text{ rate}' = \text{Flow rate} \bullet 0.80^4$$

$$\textbf{\textit{Ans.}} \quad \text{Flow rate}' = \frac{20cm^3}{\text{sec}} \bullet 0.80^4 = 8.2\frac{cm^3}{\text{sec}}$$

c) First determine the ratio of the new pressure difference to the original pressure difference.

$$\frac{\Delta P'}{\Delta P} = \frac{40cm \text{ of water}}{25cm \text{ of water}} = 1.6$$

From Poiseuille's law we know that flow rate is ***proportional*** to the pressure difference. Therefore the new flow rate will equal the original flow rate ***multiplied*** by 1.6.

$$\text{Flow rate}' = \text{Flow rate} \bullet 1.6$$

$$\textbf{\textit{Ans.}} \quad \text{Flow rate}' = 20\frac{cm^3}{\text{sec}} \bullet 1.6 = 32\frac{cm^3}{\text{sec}}$$

32d) From Poiseuilles's law,

$$\text{Flow Rate} = \frac{\Delta P \pi r^4}{8\eta L}$$

we can see that flow rate is ***directly proportional*** to both ΔP and r^4. First the ratios of new ΔP to old ΔP:

$$\frac{\Delta P'}{\Delta P} = \frac{50 \text{ cm of water}}{25 \text{ cm of water}} = 2$$

$$\frac{r'}{r} = \frac{0.20cm}{0.50cm} = 0.40$$

Because flow rate is ***directly proportional*** to both ΔP and r^4, we ***multiply***:

$$\textbf{\textit{Ans.}} \ \text{Flow rate}' = 20\frac{cm^3}{\text{sec}} \bullet 2 \bullet 0.40^4 = 1.02\frac{cm^3}{\text{sec}}$$

6.33. (II) Early on a summer day water pressure is high and produces a flow rate of 20 liters/min through a garden hose. Later in the day, when it is hot and water use is heavy throughout town, pressure drops and the flow rate through the same hose is only 8.0 liters/min. If the original pressure at the entrance of the hose was 30 m of water, <u>how large is the pressure later in the day</u>?

Notice that this problem asks us about ΔP and not about the new flow rate. Therefore, we will solve Poiseuilles's law for ΔP and then proceed as we did in earlier problems.

$$\text{Flow Rate=}\frac{\Delta P\pi r^4}{8\eta L}$$

Solving for ΔP yields our new working equation.

$$\Delta P = \frac{F.R.\bullet 8\eta L}{\pi r^4}$$

Flow rate is ***<u>proportional</u>*** to the pressure difference. First the new ratios of new flow rate to old flow rate:

$$\frac{Flow\ \text{rate}'}{Flow\ \text{rate}} = \frac{8\frac{liters}{\text{min}}}{20\frac{liters}{\text{min}}} = 0.40$$

Remember, proportional indicates that we ***multiply*** ΔP by the ratio.

Ans. $\Delta P' = \Delta P \bullet 0.40 = 30\text{m of water} \bullet 0.40 = 12\text{m of water}$

6.34. (III) If the radius of a blood vessel is reduced by cholesterol deposits to 90% of its original value, flow rate will decrease. By what factor would the pressure difference along the vessel have to be increased to get the flow rate up to its original value?

Notice that this problem asks us about ΔP and not about the new flow rate. Therefore, we will solve Poiseuilles's law for ΔP and then proceed as we did in earlier problems.

$$\text{Flow Rate=}\frac{\Delta P\pi r^4}{8\eta L}$$

Solving for ΔP yields our new working equation.

$$\Delta P = \frac{F.R.\bullet 8\eta L}{\pi r^4}$$

We are told that the new radii are 0.90 of their original size. We can see that the pressure difference is ***inversely proportional*** to r^4. Therefore the new pressure difference will equal the original pressure difference ***divided*** by 0.9^4.

Ans. $$\Delta P' = \frac{\Delta P}{0.9^4} = \frac{1\Delta P}{0.9^4} = 1.52\,\Delta P$$

Wow! If the radii are 90% of there original value, your blood pressure would have to be 1.52 times higher than when you were healthy, in order to maintain a normal blood flow rate of approximately 5 to 6 liters per minute.

6.35. (III) By what factor would the radii of the arteries have to decrease to reduce blood flow rate by a factor of 2? (Reducing by a factor of 2 means to cut the flow rate in half).

From the equation for flow rate, we can see that flow rate is ***directly proportional*** to r^4.

$$\text{Flow Rate} = \frac{\Delta P \pi r^4}{8\eta L}$$

We are told that the new flow rate is one-half of the original flow rate.

$$\frac{\text{new } \textit{flow} \text{ rate}}{\textit{original} \text{ flow rate}} = 0.50$$

Solving our Flow rate equation for r yields:

$$r = \left(\frac{F.R.\bullet 8\eta L}{\Delta P \pi}\right)^{0.25}$$

Ans. $$r' = r\bullet 0.5^{0.25} = 0.841r$$

Extra question Calculate the flow speed above which turbulent flow should occur in the aorta. Assume a diameter of 2.0 centimeter and a Reynold's number of 1900. For the density of blood, use 1.05 $\frac{g}{cm^3}$. The viscosity of blood is 0.035 $\frac{g}{cm \bullet \sec}$.

The Reynold's number equation is: $\text{Renold's number} = \frac{\rho d v}{\eta}$

Where;

ρ is the density of the fluid.

d is the diameter of the vessel
v is the velocity of the fluid
η is the viscosity of the fluid

Section 6.6

6.38. (I) The aorta is 1.0 cm in radius and the average speed of blood in the aorta is 30 cm/sec in a normal resting adult. Calculate the blood flow rate in the aorta in a) cubic centimeters per second and b) liters per minute.

a) The volume of a cylinder is equal to the area of one end multiplied by its length. We will use this fact to derive a convenient equation for flow rate.

$$Flow\ \text{rate} \equiv \frac{\text{Volume}}{\text{time}} = Area \bullet \frac{Length}{time} = Area \bullet \bar{v}$$

The area of the aorta opening is the area of a circle:

$$Area = \pi r^2 = \pi \bullet (1.0 cm)^2 = 3.14 cm^2$$

Ans. $$Flow\ \text{rate} = \text{Area} \bullet \bar{\text{v}} = 3.14\text{cm}^2 \bullet 30 \frac{cm}{\text{sec}} = 94.2 \frac{cm^3}{\text{sec}}$$

b) Convert units to liters per min:

Ans. $$94.2 \frac{cm^3}{\sec} \bullet \frac{1 liter}{1000 cm^3} \bullet \frac{60 \sec}{1 \min} = 5.65 \frac{liters}{minute}$$

Note that this blood flow rate is in the range of that for a average, resting adult.